CBCS SCHEME

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15MAT41

Fourth Semester B.E. Degree Examination, June/July 2023
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks: 80

Note: ***1. Answer FIVE full questions, choosing ONE full question from each module.***
2. Use of Statistical tables is permitted.

Module-1

1 a. Employ Taylor's series method to obtain approximate value of y at x = 0.1 and 0.2 correct to four significant figures if y satisfies the equation $\frac{dy}{dx} = 2y + 3e^x$ given that y = 0 when x = 0 taking the four terms of the Taylor's series expansion. **(05 Marks)**

b. Using Runge – Kutta method of fourth order, solve for y at x = 1.2 from $\frac{dy}{dx} = \frac{2xy + e^x}{x^2 + xe^x}$ given $x_0 = 1$, $y_0 = 0$. (take h = 0.2). **(05 Marks)**

c. Apply Milne's predictor – corrector formulae to compute y(0.4), given $\frac{dy}{dx} = xy + y^2$ and use the corrector formula.

x	0.0	0.1	0.2	0.3
y	1.0000	1.1169	1.2773	1.5049

(06 Marks)

OR

2 a. Solve the following differential equation by Euler's modified method $\frac{dy}{dx} = x + \left|\sqrt{y}\right|$, y(0) = 1 at x = 0.2 with h = 0.2. **(05 Marks)**

b. Apply Runge – Kutta method of fourth order to find an approximate value of y for x = 0.1, if $\frac{dy}{dx} = x + y^2$, given that y = 1, where x = 0 (take h = 0.1). **(05 Marks)**

c. Given that $\frac{dy}{dx} = x - y^2$ and y(0) = 1, y(0.1) = 0.9117, y(0.2) = 0.8494, y(0.3) = 0.8061 to find y(0.4) by Adams – Bashforth method. Use corrector formula twice. **(06 Marks)**

Module-2

3 a. Using Runge – Kutta method of order four, solve y″ = y + xy′, y(0) = 1, y′(0) = 0 to find y(0.2). **(05 Marks)**

b. Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials. **(05 Marks)**

c. Solve the Bessel's differential equation : $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$ leading to Bessel's function of first kind only. **(06 Marks)**

OR

4 a. Apply Milne's method to compute y(1.4) given that $2\frac{d^2y}{dx^2} = 4x\frac{dy}{dx}$ and the following table of initial values :

x	1	1.1	1.2	1.3
y	2	2.2156	2.4649	2.7514
y′	2	2.3178	2.6725	3.0657

(05 Marks)

b. With usual notation, show that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$. **(05 Marks)**

c. With usual notation, derive the Rodrigue's formula :

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}\left[(x^2-1)^n\right].$$ **(06 Marks)**

Module-3

5 a. Derive Cauchy – Riemann equations in Polar form. **(05 Marks)**

b. Evaluate $\oint_c \frac{z-3}{z^2+2z+5}dz$, where c is the circle $|z| = 1$ using Cauchy's residue theorem. **(05 Marks)**

c. Find the bilinear transformation which maps the points z = 1, i, –1 onto the points w = i, 0, – i. **(06 Marks)**

OR

6 a. Find the analytic function, whose real part is $\frac{\sin 2x}{(\cosh 2y - \cos 2x)}$. **(05 Marks)**

b. Evaluate $\oint_c \frac{e^2}{z+i\pi}dz$, where c is the circle $|z| = 2\pi$, using Cauchy's integral formula. **(05 Marks)**

c. Discuss the transformation $w = 2+\frac{1}{z}$, $z \neq 0$ with respect to the curves r = constant (≠0) and θ = constant (≠0). **(06 Marks)**

Module-4

7 a. Obtain mean and variance of the Poisson distribution. **(05 Marks)**

b. The probability mass function of a random variable x is :

x :	0	1	2	3	4	5	6
P(x) :	K	3K	5K	7K	9K	11K	13K

Find the : i) the value of K

ii) $P(x \geq 5)$, $P(3 < x \leq 6)$ and $P(x < 4)$. **(05 Marks)**

c. Find the joint distribution of x and y, which are independent random variables with the following respective distributions :

x_i	1	2
$f(x_i)$	0.7	0.3

y_j	–2	5	8
$g(y_j)$	0.3	0.5	0.2

Show that cov (x, y) = 0. **(06 Marks)**

OR

8 a. When a coin is tossed 4 times, find the probability of getting :
i) Exactly one head
ii) Atmost 3 heads
iii) At least 2 heads. **(05 Marks)**

b. The marks x obtained in mathematics by 1000 students is normally distributed with mean 78% and S.D. 11%. Determine :
i) How many students got marks above 90%?
ii) What was the highest mark obtained by the lowest 10% of students? **(05 Marks)**

Find : i) Marginal distributions f(x) and g(y)
ii) E(x) and E(y)
iii) cov(x, y) for the following joint distribution.

x \ y	–4	2	7
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
5	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

(06 Marks)

Module-5

9 a. Ten individuals are chosen at random from a population and their heights in inches are found to be 63, 63, 66, 67, 68, 69, 70, 71, 71. Test the hypothesis that the mean height of the universe is 66 inches. ($t_{0.05} = 2.262$ for 9 d.f.). **(05 Marks)**

b. The number of accidents per day (x) as recorded in a textile industry over period of 400 days is given below. Test the goodness of fit in respect to Poisson distribution of fit to the given data $\left(\chi^2_{0.05} = 9.49 \text{ for } 4\,\text{d.f}\right)$.

x	0	1	2	3	4	5
f	173	168	37	18	3	1

(05 Marks)

c. Three boys A, B, C are throwing ball to each other. A always throws the ball to B and B always throws the ball to C, C is just as likely to throw the B as to A. if C was the first person of throw the ball. Find the probabilities that after three throws :
i) A has the ball
ii) B has the ball
iii) C has the ball. **(06 Marks)**

OR

10 a. A coin is tossed 1000 times and it turns up head 540 times. Decide on the hypothesis that the coin is unbiased. **(05 Marks)**

b. Find the unique probability vector for the regular stochastic matrix :

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}.$$

(05 Marks)

c. A student's study habits are as follows :
If he studies one night, he is 60% sure not to study the next night ; on the other hard if he does not study one night, he is 80% sure to study the next night. In the long run how often does he study? **(06 Marks)**

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